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Review Article

REVIEW OF BREATHING LITERATURE ON FUZZY GAME THEORY

Arti Chouksey and Dr. Sonal Bharti

School of Engineering
Sri Satya Sai University of Technology & Medical Sciences, Sehore, India

Abstract: We survey an emerging literature on game theory for the analysis of the decision making process. The different techniques of fuzzy game theory versus their crisp prototypes are described. The properties of the crisp and fuzzy cooperative and non-cooperative games are compared.

Key words: - Fuzzy Logic, Game Theory, Fuzzy Cooperative and Non-cooperative Games

Introduction: A game is a decision-making situation with many players, each having objectives that conflict with each other. The players involved in the game usually make their decisions under conditions of risk or uncertainty. In the paper Song Q.[16] , a fuzzy approach is proposed to solve the strategic game problem in which the pure strategy set for each player is already defined. Based on the concepts of fuzzy set theory, the approach uses a multi-criteria decision-making method to obtain the optimal strategy in the game, a method which shows more advantages than the classical strategy and shows better performance for the famous “prisoner's dilemma” problem.

For Correspondence:
artijaiswal17@gmai.com.
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Classical game theory is concerned with how rational players make decisions when they are faced with known payoffs. In the past decade, Fuzzy Logic has been widely used to manage uncertainties in games. In contrary to crisp game, the fuzzy logic based game is very powerful in managing the uncertainties. In [6] Chakeri, A., employ fuzzy logic to determine the priority of a payoff to other payoffs. A new term is introduced to measure the preference of one payoff to others. A least deviation method is applied to obtain a fuzzy preference relation, and the priority for each player is calculated. In the proposed method the fuzzy payoffs, fuzzy satisfaction functions, and satisfaction degree from each payoff are defined. The calculation of similarity between satisfaction functions enables making crisp game from the fuzzy one.

In [23] the theory of fuzzy moves (TFM) is developed by the merging of theory of moves and the theory of fuzzy sets. The theory of fuzzy moves is used to make the better fuzzy moves.

To make more reasonable moves, the fuzzy sets with higher granularity for fuzzy reasoning are used. The computer simulation shows that TFM with fuzzy reasoning shows better and more reasonable performance compare to theory of moves with precise reasoning.

The theory of moves (TOM) is a type of game in which the players should decide the appropriateness of the move to be made. The fuzzy game theory of moves is presented by Barne, J.M. in [4], and the value of moves and their interrelationships are described.

In [11] is about fuzzy theory based double action model that is used to represent the auction participant's bidding wills. Using Bellman and Zadeh's concept of confluence of fuzzy decisions shows better results compare to Nash equilibrium game theory. The solved problem shows the usefulness of the proposed method in practical auctions.

In [8] considers the extension of P-cores concept in cooperative fuzzy game theory. This concept is extended from P-cores and P-stable sets to generalized P-cores, and generalized P-stable sets. The developed concept provides more rational distribution schemes. The value of generalized P-cores for cooperative fuzzy game is defined.

The extension of concept of the bargaining sets from classical game theory to non-transferable utility fuzzy game is offered in [22]. It is proven that the relation between above theories exists in both super additive non-transferable and non-transferable fuzzy games. The importance and significant contribution of the proposed concept is described.

Basic properties, presented in [5], define two-stage production games. For the solving of the production games the hybrid algorithm, which is the combination of genetic algorithm (GA), neural network (NN) and approximating method, is designed. The distribution games example solved shows the feasibility of the new algorithm.

Usually in conventional game theory each player has a strategy with well-defined outcome. Nowadays the complexity of problems in many areas does not reflect the correctness of the initial assumption to be accepted, because a crisp payoff is defined with a big difficulty. In [12] offered fuzzy numbers are used to incorporate the results of strategies. To define payoffs, the creditability measure is used.

In [10] describes the cooperative fuzzy games which deal with the fuzzy coalitions and infinite players. The natural class of fuzzy games with Choquet integral has several rational properties such as convexity, supper additive and monotonicity.

Sometimes using crisp game theory does not lead to effective modelling the incorporation some of the subjective attitudes of the decision makers because of the vague and ambiguous types of information. In [2] the fuzzy approach is presented to solve the Prisoner's Dilemma in which the decision makers should decide whether or not to cooperate. The fuzzy procedure considers subjective attitudes of the decision makers to act under uncertain and risky types of situations.

In case of incompleteness, ambiguity, vagueness and impreciseness of the situations the decision maker can't model a conflict to get a feasible preference, and it affects the overall equilibria to be predicted. In [3] developed method is intended for uncertainty modelling to resolve the conflict in the preferences of the decision maker. In order to illustrate the importance of the developed approach to find the realistic equilibria, the fuzzy preference methodology is applied on prisoner's dilemma problem of game theory.

Sun., Linlin, in [14] presents a new model of interval fuzzy cooperative games with Choquet integral form. The relationship between fuzzy convex form of Choquet integral and interval Shapley value is described. It is mentioned that improved Shapley value is very important in fuzzy games with interval fuzzy number.

In [9] some possible two-agent decision making problems are represented which involve perceptions of one agent about the other agent.

The importance of defining information links between the agents is explained. The case, when players have fuzzy but close to true criteria, is investigated. It is shown that both players expect actual values from their calculated strategies similar to while making their fuzzy hypotheses.
A new approach proposed by Sharma, R. in [20] is used to incorporate a hybrid game strategy in Markov-game-based fuzzy control. The universal controller is designed to show an ability of a good performance against disturbance and environment variations. The hybrid control based on experiential information obtains reasonable performance against above variations in Markov-game-based control.
In [19] the fuzzy linguistic preference relation in game theory is described. The priorities of Nash equilibrium are investigated. In order to compare fuzzy variable, two measures are represented by using fuzzy extension principle.
As it is known in game theory, the player's main task consists in maximizing their payoffs. It is difficult to perform this task in the presence of fuzzy and uncertain natures. [7] Considers a novel approach to analyze the games with fuzzy payoffs method to find pure strategy Nash equilibrium. The priorities of payoffs are determined by ranking fuzzy numbers.
Li, K.W., in [13] two different fuzzy methods for the studying 2×2 game model are proposed. In the first method the multi-criteria decision analysis is investigated to obtain optimal strategies of the players. In the second method the application of the theory of fuzzy moves (TFM) for the Chicken game is considered. The importance of using theory of moves consists in the presence of factors to look ahead to improve the decision making process. It is also observed that using above fuzzy methods provide better result for the game of Chicken and demonstrate their effectiveness in the presence of uncertain and vague information.
The new non-cooperative model of a normal form game is introduced in [15]. Bellman and Zadeh's principle of a decision theory is extended to game theory. The conditions for the existence of equilibrium are investigated.
Most Internet transactions are modelled using terms of traditional game theory. Price negotiations, competition for customers, and online auctions can be given as examples. In case of dealing with uncertain values, these games become examples of fuzzy game theory. In [18] proposed fuzzy approach for the game theory is applied to consider some specific peculiarities of e-commerce.
The development of negotiation model in electronic commerce has become an important issue to implement trade-off. In [24] the fuzzy set theory based negotiation model is established which is used to solve the following problems: The normalization process is performed for the goals to define the weight vectors and payoff matrix; the negotiation of multi-goals is realized; and the strategy for the negotiation process is set.
The classical game theory method is not appropriate for using in uncertain environment in which most negotiation processes for the development of E-Commerce systems take place. For this reason in [1] the fuzzy logic based approach in game theory is proposed that overcomes the complexity of negotiation process in E-Commerce system. The implementation of the above method shows better performance to achieve benefits in negotiation parties.

Crisp Cooperative Game versus Fuzzy Cooperative Game

Crisp Cooperative game: Definition 2.1.1:- A cooperative game in characteristic function form is an ordered pair $\langle N, \vartheta \rangle$, consisting of the player set N and the characteristic function

$$\vartheta : 2^N \rightarrow R \quad \text{with } \vartheta(\phi) = 0$$

N be a non-empty finite set of agents who consider different cooperation possibilities. Each subset $S \subset N$ is referred to as a *crisp coalition*. The set N is called the grand coalition and ϕ is called the empty coalition.

Cooperative game is a coalitional game that may contain finite number of participators who agree to coordinate their strategies to optimize payoff of the players. The payoff of the game is determined by the combination of the strategies. The target of the game should satisfy the player's objective which is required from the game [11].

The target of each player in the coalition is to maximize his/her own outcomes and the other target is to maximize the outcomes of the other players in the coalition. These coalitions are mostly important in political science and international relations. For example, assuming that the players are several parties in parliament and each of these parties has different degree of power depending on the number of seats they have for the members of the party [21].

Suppose there are two companies A and B. These companies should decide whether to cooperate or not to cooperate according to the payoffs given in Figure 1:

	Company 2		
Company 1		Cooperation	Non-Cooperation
	Cooperation	(6 , 6)	(9 , 2)
	Non-Cooperation	(2 , 9)	(4 , 4)

Figure 1: Crisp payoff Matrix of a Cooperative Game

There are totally four possible situations to deal with:

1) If company 1 cooperates, then it is better for company 2 to cooperate.
2) If company 1 does not cooperate, then it is better for company 2 to cooperate.
3) If company 2 cooperates, then it is better for company 1 to cooperate.
4) If company 2 does not cooperate, then it is better for company 1 to cooperate.

So the best decision is reached when both the companies 1 and 2 decide to cooperate.

Most crisp cooperative games can be transferred into fuzzy form. In contrast to crisp cooperative game in which the players take part in a game fully or don't take part at all, fuzzy cooperative games are represented by the partial coalition between players in which the levels of their participation are taken from the interval [0, 1]. The real valued function used in fuzzy game theory can assign real values to each coalition [17].

2.2 Fuzzy Cooperative Game

Definition 2.2.1:- A fuzzy coalition is a vector $s \in [0,1]^N$.

Definition 2.2.1:- A cooperative fuzzy game with player set N is a map $\vartheta: F^N \to R$ with the property $v(e^\phi) = 0$.

Example 2.2.1:- Let $\vartheta \in FG^{\{1,2\}}$ be defined by

$$\vartheta(s_1, s_2) = \begin{cases} 1 \text{ if } & s_1 \geq \frac{1}{2}, \quad s_2 \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

for each $s = (s_1, s_2) \in \mathcal{F}^{\{1,2\}}$. This game corresponds to a situation is which only coalitions with participation levels of the players of at least $\frac{1}{2}$ are winning, and all other coalitions are losing.

We consider the cooperative fuzzy game with two players. If we consider a fuzzy cooperative game in which two players are involved, then in order to decide whether to cooperate one should take into account the values of the participation levels of both the players that are at least 1/2. [17]:

$$\vartheta(s_1, s_2) = \begin{cases} 1 \text{ if } & s_1 \geq \frac{1}{2}, \quad s_2 \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots \quad 2.1$$

Where s_1 and s_2 are the participation levels of the players 1 and 2, respectively.

Let's illustrate the above participation levels of a fuzzy cooperative game in example. In Figure 2 each cell represents levels of participation of two players 1 and 2 involved in a game:

	Company 2		
Company 1		Cooperation	Non-Cooperation
	Cooperation	(0.7 , 0.7)	(0.5 , 0.2)
	Non-Cooperation	(0.2, 0.5)	(0.1 , 0.1)

Figure 2: Fuzzy Matrix of a Cooperative Game

As it can be seen from the table, the optimal solution of the problem (according to the values of the participation levels of the players) is reached when both the players agree to cooperate.

Crisp Non -Cooperative Game versus Fuzzy Non -Cooperative Game

In non-cooperative game the players analyze their strategic choices in decision making process. There is no agreement between players before the game, i.e. neither of the players agrees to cooperate. In non-cooperative game the players are acting in self-interest. Each player chooses the best outcome for him/her no matter what another player undertakes to act.

Non-cooperative game theory is mostly applied in bargaining which produce a specific process that determine who would get an offer for the choices at a specific time [12].

According to the payoffs of companies 1 and 2 represented in Figure 3, it is possible to see that if either the company 1 or 2 decides to cooperate, their payoffs get less if they choose the option of non-cooperation. And the best outcome for both the companies A and B is reached when the decision is not to cooperate.

Using the formula (2.1), it is possible to apply fuzzy approach to non-cooperative game. As it is seen from the Figure 4, the optimal decision undertaken is reached when both the companies decide not to cooperate (according to the values of participation levels).

Company 1	Company 2		
		Cooperation	Non-Cooperation
	Cooperation	(3,3)	(2,6)
	Non-Cooperation	(6,2)	(8,8)

Figure 3: Crisp payoff Matrix of a Non-Cooperative Game

Company 1	Company 2		
		Cooperation	Non-Cooperation
	Cooperation	(0.3,0.3)	(0.2,6)
	Non-Cooperation	(0.6,0.2)	(0.8,0.8)

Figure 4: Fuzzy Matrix of a Non- Cooperative Game

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