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Original Research Article

S-PATH DOMINATION IN SHADOW DISTANCE GRAPHS

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Abstract: Let G = (V,E) be a simple connected and undirected graph. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A dominating set of G is called a s-path dominating set of G $(3 \le s \le diamG)$ if every path of length s in G has at least one vertex in this dominating set. We denote a s-path dominating set by D_{p_s}. The s-path domination number of G denoted by $\gamma_{p_s}(G)$ is the minimal cardinality taken over all s - path dominating sets of G. In this paper, we determine s - path domination number of the shadow distance graph of the path graph with specified distance sets.

Keywords: - Dominating set, vertex domination number, s-path domination number, Minimal vertex dominating set.

Introduction: By a graph $G=(V,E)$ we mean a finite undirected graph without loops and multiple edges. A subset D of V is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number or vertex domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A vertex v in a graph G dominates the vertices in its closed neighbourhood N(v), that is, v is said to dominate itself and each of its neighbours.

A dominating set of G is called a s-path dominating set of G $(3 \le s \le diamG)$ if every path of length s in G has atleast one vertex in this dominating set. We denote a s-path dominating set by D_{p_s}.The s-path domination number of G denoted by $\gamma_{p_s}(G)$ is the minimal cardinality taken over all s-path dominating sets of G. By definition every s-path dominating set is a dominating set but the converse is not true. Also it follows that $|D| \le |D_{p_s}|$ and hence $|\gamma(G)| \le |\gamma_{p_s}(G)|$.

Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The

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distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u,v) \in D_s$.

The shadow distance graph of G, denoted by $D_{sd}(G, D_s)$ is constructed from G with the following conditions:

i) consider two copies of G say G itself and G'

ii) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u' \in V(G')$ (second copy)

iii) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G')$

iv) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G') \cup E_{ds}$ where E_{ds} is the set of all edges between two distinct vertices $u \in V(G)$ and $v' \in V(G')$ that satisfy the condition $d(u,v) \in D_s$ in G.

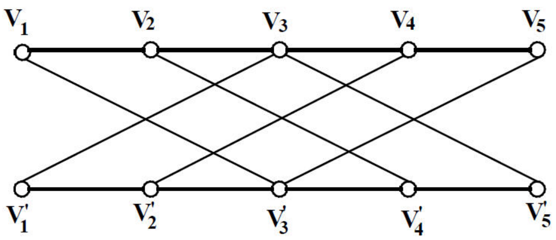

Figure 1. The graph $D_{sd}(P_5, \{2\})$

Main Results

Theorem 2.1. If G is a graph with no isolated vertices, then $\gamma(G) \le \gamma_{p_3}(G) \le \frac{n}{2}$

Proof: Let D_{p_s} is a minimal dominating set of G. Every vertex in D_{p_s} adjacent with at least one vertex in V- D_{p_s}. Hence V- D_{p_s} is a dominating set and $\gamma(G) \le \gamma_{p_s}(G) \le \min\left\{\left|D_{p_s}\right|, \left|V - D_{p_s}\right|\right\} \le \frac{n}{2}$.

Theorem 2.2.For any graph G,

$$\gamma(G) \le \gamma_{p_s}(G) \le \left\lceil \frac{n+1-(\delta(G)-1)\frac{\Delta(G)}{\delta(G)}}{2} \right\rceil$$

Proof : The upper bound is immediate.

Theorem 2.3. For any graph G, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \le \gamma(G) \le \gamma_{p_s}(G)$

Proof: Let D_{p_s} be s-path dominating set of G. Each vertex dominates at most itself and $\Delta(G)$ other vertices. Hence the result.

The following results are immediate from the definition

Theorem 2.4. Let $n \ge 3$. Then $\gamma_{p_s}(P_n) = \left\lceil \frac{n}{3} \right\rceil$, $3 \le s \le diamP_n$

We recall the following result related to $\gamma(G)$.

Theorem 2.5. [5] A dominating set D is a minimal dominating set if and only if for each vertex v in D, one of the following condition holds:

i) v is an isolated vertex of D

ii) there exists a vertex u $\in$ V-D such that N(u) $\cap$ D = $\{v\}$

An analogous result related to s-path domination is stated below;

Theorem 2.9. A dominating set D_{p_s} is a minimal dominating set if and only if for each vertex v in D_{p_s}, one of the following condition holds:

i) v is an isolated vertex of D_{p_s}

ii) there exists a vertex u $\in$ V- D_{p_s} such that N(u) $\cap$ D_{p_s} = $\{v\}$

We first provide below the results for vertex domination number of the shadow distance graph of the path graph with specified distance sets.

Theorem 2.10. Let $n \ge 5$. Then

$$\gamma(D_{sd}\{P_n, \{2\}\}) = 2\left\lceil \frac{n}{5} \right\rceil.$$

Proof : Consider two copies of P_n, one P_n itself and other denoted by $P_n^{'}$. Let $v_1, v_2, ... v_n$ be the vertices of P_n and let $v_1^{'}, v_2^{'}, ... v_n^{'}$ be the vertices of $P_n^{'}$. Let $e_1, e_2, ... e_{n-1}$ be the edges of the first copy P_n and $e_1^{'}, e_2^{'}, ... e_{n-1}^{'}$ be the edges of the second copy $P_n^{'}$, where $e_i = (v_i, v_{i+1}), e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for i = 1,2,…n-1.

Let $G = (D_{sd}\{P_n, \{2\}\})$.

Then $|V(G)| = 2n, |E(G)| = 4n-6$ and

$$E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{j,\{j+2\}^{'}}\} \cup \{e_{k,\{k-2\}^{'}}\}$$

where $1 \le i \le n-1, 1 \le j \le n-2, 3 \le k \le n$.

Let $n \ge 6$.

Consider the set D = $V_1 \cup V_2$ where

$$V_1 = \{v_{5i-2}\} \cup \{v_{5i-2}^{'}\}, 1 \le i \le \left\lceil \frac{n}{5} \right\rceil - 1,$$

$$V_2 = \begin{cases} \{v_n, v_n^{'}\}, & n \equiv 1,2,3 (\bmod 5) \\ \{v_{n-1}, v_{n-1}^{'}\}, & n \equiv 4 (\bmod 5) \\ \{v_{n-2}, v_{n-2}^{'}\}, & n \equiv 0 (\bmod 5) \end{cases}$$

This set D is a minimal dominating set with minimum cardinality since for any vertex v ∈ D , D- {v} is not a dominating set. Thus, some vertex u in V-D is not dominated by any vertex in D∪ {v}. Now either u=v or u ∈ V-D. If u=v, then v is an isolated vertex of D. If u∈ V-D and u is not dominated by D - {v}, but is dominated by D, then u is adjacent only to vertex v in D, i.e N(v) ∪ D = {v}.

This implies that the set D described above is of minimum cardinality and since $|D| = 2\left\lceil \frac{n}{5} \right\rceil$, it follows that

$$\gamma(D_{sd}\{P_n, \{2\}\}) = 2\left\lceil \frac{n}{5} \right\rceil.$$

Theorem 2.11. Let $n \ge 5$. Then

$$\gamma(D_{sd}\{P_n, \{3\}\}) = 2\left\lceil \frac{n+2}{5} \right\rceil.$$

Proof : Let $G = (D_{sd}\{P_n, \{3\}\})$ We consider the vertex set of G as in Theorem 2.10. and edge set

$$E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{j,\{j+3\}^{'}}\} \cup \{e_{\{k-3\}^{'},k}\}$$

where $1 \le i \le n-1, 1 \le j \le n-3, 1 \le k \le n$. Clearly $|V(G)| = 2n, |E(G)| = 4n-8$.

Let n $\ge$ 5.

Consider the set D = $V_1 \cup V_2$ where

$$V_1 = \{v_{5i-3}\} \cup \{v_{5i-3}^{'}\}, 1 \le i \le \left\lceil \frac{n-3}{5} \right\rceil,$$

$$V_2 = \begin{cases} \{v_n, v_n^{'}\}, & n \equiv 2,3,4 (\bmod 5) \\ \{v_{n-1}, v_{n-1}^{'}\}, & n \equiv 0,1 (\bmod 5) \end{cases}$$

This set D is a minimal dominating set with minimum cardinality since for any vertex v ∈ D , D- {v} is not a dominating set. Thus, some vertex u in V-D is not dominated by any vertex in D∪ {v}. Now either u=v or u ∈ V-D. If u=v, then v is an isolated vertex of D. If u∈ V-D and u is not dominated by D - {v}, but is dominated by D, then u is adjacent only to vertex v in D, i.e N(v) ∪ D = {v}.

This implies that the set D described above is of minimum cardinality and since $|D| = 2\left\lceil \frac{n+2}{5} \right\rceil$, it follows that $\gamma(D_{sd}\{P_n, \{3\}\})$ = $2\left\lceil \frac{n+2}{5} \right\rceil$.

Hence the proof.

Theorem 2.12. Let n $\ge$ 5. Then

$$\gamma_{p_3}((D_{sd}\{P_n, \{2\}\})) = \begin{cases} 4, & n = 5 \\ 6, & n = 6,7 \\ 2\left\lceil \frac{n}{2} \right\rceil - 2, & n \ge 8 \end{cases}$$

Proof : Let $G = (D_{sd}\{P_n, \{2\}\})$. We consider the vertex set and edge set of G are as in Theorem 2.10.

For n=5, the set D_{p_3} = $\{v_3, v_4, v_3^{'}, v_4^{'}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =4.

For n=6, the set D_{p_3} = $\{v_3, v_4, v_6, v_3^{'}, v_4^{'}, v_6^{'}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

For n=7, the set D_{p_3} = $\{v_3, v_4, v_7, v'_3, v'_4, v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

For n=8, the set D_{p_3} = $\{v_3, v_4, v_7, v'_3, v'_4, v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

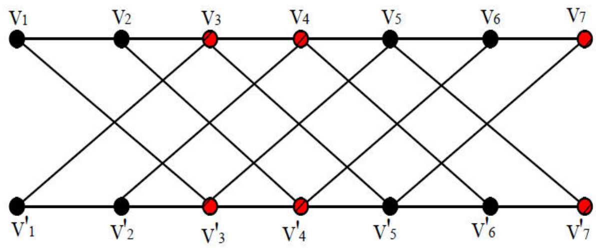

Figure 2. The graph $\gamma_{p_3}(D_{sd}(P_7,\{2\})) = 6$

Let $n \geq 9$.

Consider the set D_{p_3} =

$$\begin{cases} \{v_{4j-1}\} \cup \{v_{4j}\} \cup \{v'_{4j-1}\} \cup \{v'_{4j}\}, & n \equiv 1,2 (\bmod 4) \\ \{v_{4j-1}\} \cup \{v_n\} \cup \cup \{v_{4j}\} \cup \{v'_{4j-1}\} \cup \{v'_n\} \cup \{v'_{4j}\}, & n \equiv 3 (\bmod 4) \\ \{v_{4j-1}\} \cup \{v_{n-1}\} \cup \cup \{v_{4j}\} \cup \{v'_{4j-1}\} \cup \{v'_{n-1}\} \cup \{v'_{4j}\}, & n \equiv 0 (\bmod 4) \end{cases}$$

$$\text{where} \begin{cases} 1 \leq j \leq \left\lfloor \frac{n}{4} \right\rfloor, & n \equiv 1,2 (\bmod 4) \\ 1 \leq j \leq \left\lfloor \frac{n}{4} \right\rfloor, & n \equiv 3 (\bmod 4) \\ 1 \leq j \leq \frac{n}{4} - 1, & n \equiv 0 (\bmod 4) \end{cases}$$

This set D_{p_3} is aminimal dominating sets with minimum cardinality since for any vertex v ∈ D_{p_3} , D_{p_3} - {v} is not a 3-path dominating set. Thus, some vertex u in V-D_{p_3} is not dominated by any vertex in $D_{p_3} \cup$ {v}. Now either u=v or u ∈ V-D_{p_3}. If u=v, then v is an isolated vertex of D_{p_3}. If u∈ V-D_{p_3} and u is not dominated by D_{p_3} - {v}, but is dominated by D_{p_3}, then u is adjacent only to vertex v in D_{p_3},i.e N(v) ∪ D_{p_3} = {v}.

This implies that the set D_{p_3} described above is of minimum cardinality and since $|D_{p_3}| = 2\left\lceil \frac{n}{2} \right\rceil - 2$ it follows that

$\gamma_{p_3}((D_{sd}\{P_n,\{2\}\}))$ = $2\left\lceil \frac{n}{2} \right\rceil - 2$.

Hence the proof.

Theorem2.13. Let n≥5.Then $\gamma_{p_3}(D_{sd}\{P_n,\{3\}\})$

$$= \begin{cases} 4, & n = 5 \\ 6, & n = 6 \\ 2\left\lceil \frac{n}{2} \right\rceil, & n \geq 7 \end{cases}$$

Proof: : Let $G = (D_{sd}\{P_n,\{3\}\})$ We consider the vertex set and edge set of G are as in Theorem 2.11.

For n=5, the set D_{p_3} = $\{v_2, v_4, v'_2, v'_4\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =4.

For n=6, the set D_{p_3} = $\{v_2, v_4, v_5, v'_2, v'_4, v'_5\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_3}(G)$ =6.

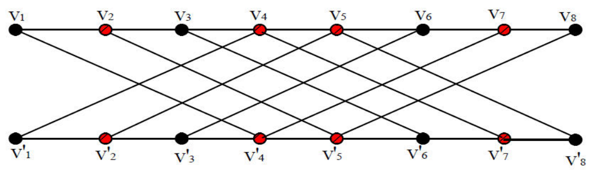

Figure 3. The graph $\gamma_{p_3}(D_{sd}(P_8,\{3\}))=8$

Let n ≥ 7.

Consider

$$D_{p_3} = V_1 \cup V_2,\ where\ V_1=\{v_2,v_4,v'_2,v'_4\}, V_2=\{v_{2j+3}\}\cup\{v'_{2j+3}\}, 1\leq j\leq \left\lceil \frac{n}{2}\right\rceil -2$$

This set D_{p_3} is aminimal dominating sets with minimum cardinality since for any vertex v ∈ D_{p_3} , D_{p_3} - {v} is not a 3-path dominating set. Thus, some vertex u in V-D_{p_3} is not dominated by any vertex in $D_{p_3} \cup$ {v}. Now either u=v or u ∈ V-D_{p_3}. If u=v, then v is an isolated vertex of D_{p_3}. If u∈ V-D_{p_3} and u is not dominated by D_{p_3} - {v}, but is dominated by D_{p_3}, then u is adjacent only to vertex v in D_{p_3}, i.e N(v) ∪ D_{p_3} = {v}.

This implies that the set D_{p_3} described above is of minimum cardinality and since $|D_{p_3}| = 2\left\lceil \frac{n}{2}\right\rceil$ it follows that $\gamma_{p_3}(D_{sd}\{P_n,\{3\}\}) = 2\left\lceil \frac{n}{2}\right\rceil$.

Hence the proof.

Theorem2.14. $\gamma_{p_4}((D_{sd}\{P_n,\{2\}\})) =$

$$\begin{cases} \left\lceil \frac{n}{3}\right\rceil, & 6\leq n\leq 10 \\ 2\left\lfloor \frac{n+(2j+2)}{3}\right\rfloor, & 6j+5\leq n\leq 7j+10,\ j\geq 1 \end{cases}$$

Proof: Let $G=(D_{sd}\{P_n,\{2\}\})$. We consider the vertex set and edge set of G are as in Theorem 2.10.

For n=6, the set D_{p_4} = $\{v_3,v_4,v'_3,v'_4\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$ =4.

For n=7, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$ =6.

For n=8, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$ =6.

For n=9, the set D_{p_4} = $\{v_3,v_4,v_7,v'_3,v'_4,v'_7\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$ =6.

For n=10, the set D_{p_4} = $\{v_3,v_4,v_7,v_{10},v'_3,v'_4,v'_7,v'_{10}\}$ is a minimal vertex dominating set with minimum cardinality and hence $\gamma_{p_4}(G)$ =8.

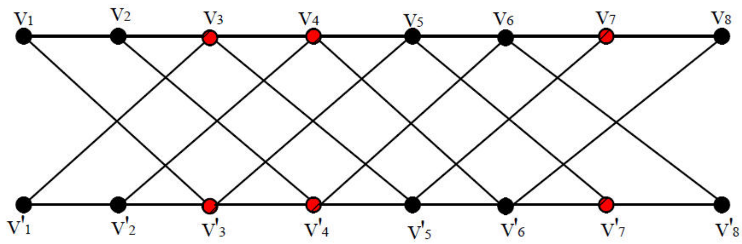

Figure 4. The graph $\gamma_{p_4}(D_{sd}(P_8,\{2\}))=6$

Let n ≥ 11.

Consider $D_{p_4} = V_1 \cup V_2 \cup V_3$,.

where

$$V_1=\{v_{7j-4},v_{7j-3}\}\cup\{v'_{7j-4},v'_{7j-3}\}\cup\{v_n,v'_n\}\cup\{v_{7j},v'_{7j}\},\ n\equiv 3(\bmod 7),1\le j\le\left\lfloor\frac{n}{7}\right\rfloor$$

$$V_2=\{v_{7i-4},v_{7i-3}\}\cup\{v'_{7i-4},v'_{7i-3}\}\cup\{v_{7j},v'_{7j}\},n\equiv 0,1,2(\bmod 7),1\le i\le\left\lfloor\frac{n}{7}\right\rfloor$$

$$V_3=\{v_{7k-4},v_{7k-3}\}\cup\{v'_{7k-4},v'_{7k-3}\}\cup\{v_{7k},v'_{7k}\},n\equiv 4,5,6(\bmod 7),1\le k\le\left\lceil\frac{n}{7}\right\rceil$$

This set D_{p_4} is aminimal dominating sets with minimum cardinality since for any vertex v $\in$ D_{p_4} , D_{p_4} - {v} is not a 4-path dominating set. Thus, some vertex u in V-D_{p_4} is not dominated by any vertex in $D_{p_4} \cup$ {v}. Now either u=v or u $\in$ V-D_{p_4}. If u=v, then v is an isolated vertex of D_{p_4}. If u$\in$ V-D_{p_4} and u is not dominated by D_{p_4} - {v}, but is dominated by D_{p_4}, then u is adjacent only to vertex v in D_{p_4},i.e N(v) $\cup$ D_{p_4} = {v}.

This implies that the set D_{p_4} described above is of minimum cardinality and since

$$|D_{p_4}| = 2\left\lfloor\frac{n+(2j+2)}{3}\right\rfloor,\ 6j+5\le n\le 7j+10,\ j\ge 1,$$

it follows that

$$\gamma_{p_4}((D_{sd}\{P_n,\{2\}\})) = 2\left\lfloor\frac{n+(2j+2)}{3}\right\rfloor,\ 6j+5\le n\le 7j+10,\ j\ge 1.$$

Hence the proof.

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References

1. U. Vijayachandra Kumar and R. Murali, “Edge Domination in Shadow distance Graphs ”, International journal of Mathematics and its Applications,Volume 4,Issue 2-D (2016), pp. 125 - 130 Volume 13,N0. 1. (2016), pp . 125 - 130
2. Gross.J.T and Yellen.J,“Graph theory and it's Applications ”, 2nd ed, Bocaraton, FL. CRC press2006
3. Pemmaraju.S and Skiena.S, “ Cycles, Stars and Wheels ” in Computational Discrete Mathematics, Cambridge University press pp. 248-249,2003.
4. S.T.Hedetniemi and R.C.Laskar, “Bibliography on domination in graphs and some basic definitions of domination parameters,” Discrete Mathematics, Vol. 86, No.1–3, pp.257–277, 1990
5. V.R.Kulli, “Theory of domination in graphs", Vishwa International Publications, 2013
6. S.R.Jayaram,“Line domination in graphs", Graphs Combin.3, pp. 357-363, 1987
7. Frank Harary, “Graph Theory", Addison - Wesley Publications, 1969.