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Original Research Article

LINEAR PRIME LABELING OF SOME PATH RELATED DI GRAPHS

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Abstract: Linear prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the direct edges with twice the value of the terminal vertex plus value of the initial vertex. The greatest common incidence number of a vertex (***gcin***) of in degree greater than one is defined as the greatest common divisor of the labels of the incident edges[5]. If the ***gcin*** of each vertex of in degree greater than one is one, then the graph admits linear prime labeling[6]. Here we investigate some path related di graphs for linear prime labeling.

Keywords: Graph labeling, linear, prime labeling, prime graphs, di graphs, path.

Introduction: All graphs in this paper are finite and directed. The direction of the edge is from v_i to v_j iff $f(v_i) < f(v_j)$. Let G = (V, E) be the graph with $|V| = p$ and $|E| = q$. Here we label the vertices with first p-1 whole numbers and edges with a linear function. Here we proved that direct path, direct middle graph of a path, direct total graph of a path, direct duplicate graph of a path, direct two tuple graph of a path, direct Z – graph of a path, direct shadow graph of a path and direct splitting graph are di linear prime labeled graphs.

All definitions, figures and basic results are taken from [1], [2],[3] and [4].

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Definition 1.1 Let G be a graph with p vertices and q edges. The greatest common divisor of the labels of the edges incident on a vertex is defined as the greatest common incidence number(***gcin***) of that vertex.

Definition 1.2 If each edge of a graph has a direction**,** then the graph is called di graph

Definition 1.3 The number of edges incident on a vertex in a di graph is called the in degree of that vertex.

Main Results

Definition 2.1 A di graph G with p vertices and q edges is said to admit linear prime labeling if it satisfy the following three conditions:

1. Vertices are labeled with first p-1 whole numbers.
2. Edges are labeled with sum of the label of the initial vertex of the edge and twice the label of the terminal vertex of the edges.

3. Greatest common incidence number of each vertex of in degree greater than one is one.

Definition 2.2 A di graph which admits linear prime labeling is called linear prime di graph.

Theorem 2.1 Direct path P_n (n >2) admits linear prime labeling.

Proof: Let G = P_n and let $v_1,v_2,---,v_n$ are the vertices of G.

Here |V(G)| = n and |E(G)| = n-1.

Define a function f : V → {0,1,2,---,n-1} by

$f(v_i) = i-1$, i = 1,2,---,n

Clearly f is a bijection.

f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_j\ v_{j+1})$= 3j-1, $1 \le j \le n-1$

Clearly f_{lpl}^{*} is an injection.

In degree of each vertex is less than 2.

Hence P_n , admits linear prime labeling.

Example 2.1 G = P_5.

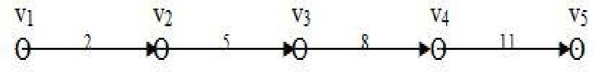

fig – 2.1

Theorem 2.2 Direct middle graph of Path P_n (n >2) admits linear prime labeling.

Proof: Let G = M{P_n} and let $v_1,v_2,---,v_{2n-1}$ are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-4.

Define a function f : V → {0,1,2,---,2n-2} by

$f(v_i) = i-1$, i = 1,2,---,2n-1

Clearly f is a bijection.

f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{2j-1}\ v_{2j})$= 6j-4, $1 \le j \le n-1$.

$f_{lpl}^{*}(v_{2j}\ v_{2j+1})$= 6j-1, $1 \le j \le n-1$.

$f_{lpl}^{*}(v_{2j}\ v_{2j+2})$= 6j+1, $1 \le j \le n-2$.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{2j+2})

= gcd of {$f_{lpl}^{*}(v_{2j}\ v_{2j+2})$, $f_{lpl}^{*}(v_{2j+1}\ v_{2j+2})$}

= gcd of {6j+1, 6j+2}

= 1, $1 \le j \le n-2$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence M{P_n}, admits linear prime labeling.

Example 2.2 G = M(P_4).

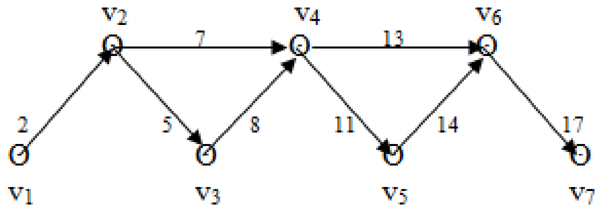

fig – 2.2

Theorem 6.2.3 Direct total graph of Path P_n (n >2) admits linear prime labeling.

Proof: Let G = T{P_n} and let $v_1,v_2,---,v_{2n-1}$ are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 4n-5.

Define a function f : V → {0,1,2,---,2n-2} by

$f(v_i) = i-1$, i = 1,2,---,2n-1

Clearly f is a bijection.

f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{2j-1}\ v_{2j})$ = 6j-4, $1 \le j \le n-1$.

$f_{lpl}^{*}(v_{2j-1}\ v_{2j+1})$ = 6j-2, $1 \le j \le n-1$.

$f_{lpl}^{*}(v_{2j}\ v_{2j+1})$ = 6j-1, $1 \le j \le n-1$.

$f_{lpl}^{*}(v_{2j}\ v_{2j+2})$ = 6j+1, $1 \le j \le n-2$.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{2j+2})

= gcd of {$f_{lpl}^{*}(v_{2j}\ v_{2j+2})$, $f_{lpl}^{*}(v_{2j+1}\ v_{2j+2})$}

= gcd of {6j+1, 6j+2}

= 1,

$1 \le j \le n-2$.

gcin of (v_{2j+1})

= gcd of {$f_{lpl}^{*}(v_{2j-1}\ v_{2j+1})$, $f_{lpl}^{*}(v_{2j}\ v_{2j+1})$}

= gcd of {6j-2, 6j-1}

= 1,

$1 \le j \le n-1$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence T{P_n}, admits linear prime labeling.

Example 2.3 G = T(P_4).

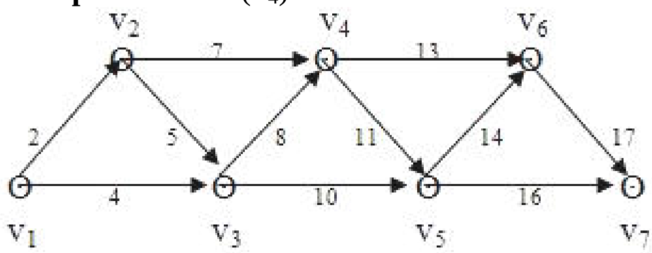

fig – 2.3

Theorem 6.2.4 Direct duplicate graph of path P_n (n > 2) admits linear prime labeling.

Proof: Let G = D{P_n} and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n-2.

Define a function f : V → {0,1,2,---,2n-1} by

$$f(v_i) = i-1 , i = 1,2,---,2n$$

Clearly f is a bijection.

f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{2j}\ v_{2j+1}) = 6j-1, \quad 1 \le j \le n-1$

$f^*_{lpl}(v_{2j-1}\ v_{2j+2}) = 6j, \quad 1 \le j \le n-1$

Clearly f^*_{lpl} is an injection.

In degree of each vertex is less than 2.

Hence P_n , admits linear prime labeling.

Example 2.4 G = D{P_4}.

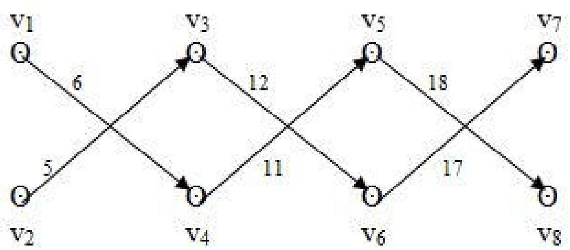

fig – 2.4

Theorem 2.5 Direct 2- tuple graph of path P_n (n > 2) admits linear prime labeling.

Proof: Let G = $T^2(P_n)$ and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-2.

Define a function f : V → {0,1,2,---,2n-1} by

$$f(v_i) = i-1 , i = 1,2,---,2n$$

Clearly f is a bijection.

f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{2j-1}\ v_{2j}) = 6j-4, \quad 1 \le j \le n.$

$f^*_{lpl}(v_{2j-1}\ v_{2j+1}) = 6j-2, \quad 1 \le j \le n-1$

$f^*_{lpl}(v_{2j}\ v_{2j+2}) = 6j+1 \quad 1 \le j \le n-1.$

Clearly f^*_{lpl} is an injection.

gcin of (v_{2j+2})

= gcd of {$f^*_{lpl}(v_{2j}\ v_{2j+2})$, $f^*_{lpl}(v_{2j+1}\ v_{2j+2})$}

= gcd of {6j+1, 6j+2}

= 1,

$1 \le j \le n-1$

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $T^2(P_n)$, admits linear prime labeling.

Example 2.5 G = $T^2(P_4)$.

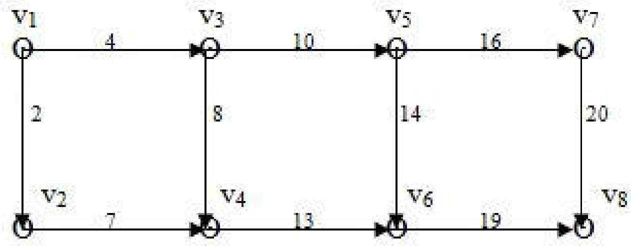

fig – 2.5

Theorem 2.6 Direct Z- graph of path P_n (n > 2) admits linear prime labeling.

Proof: Let G = Z(P_n) and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,---,2n-1} by

$$f(v_i) = i-1 , i = 1,2,---,2n$$

Clearly f is a bijection.

f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{2j-1}\ v_{2j+1}) = 6j-2, \quad 1 \le j \le n-1$

$f^*_{lpl}(v_{2j}\ v_{2j+1}) = 6j-1 \quad 1 \le j \le n-1.$

$f^*_{lpl}(v_{2j}\ v_{2j+2}) = 6j+1 \quad 1 \le j \le n-1.$

Clearly f^*_{lpl} is an injection.

gcin of (v_{2j+1})

= gcd of {$f^*_{lpl}(v_{2j-1}\ v_{2j+1})$, $f^*_{lpl}(v_{2j}\ v_{2j+1})$}

= gcd of {6j-2, 6j-1}

= 1, $\quad 1 \le j \le n-1.$

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence Z(P_n), admits linear prime labeling.

Example 2.6 G = Z(P_4).

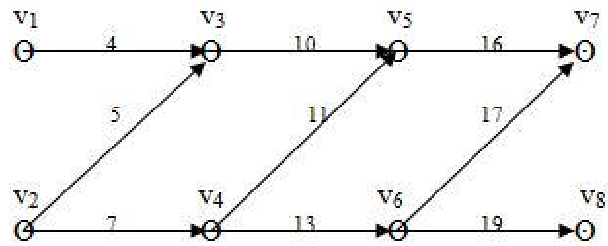

fig – 2.6

Theorem 2.7 Direct shadow graph of path P_n (n > 2) admits linear prime labeling.

Proof: Let G = $D_2(P_n)$} and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 4n-4.

Define a function f : V → {0,1,2,---,2n-1} by

$f(v_i) = i-1$, i = 1,2,---,2n

Clearly f is a bijection.

f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{2j-1}\, v_{2j+1}) = 6j-2, \quad 1 \le j \le n-1.$

$f_{lpl}^*(v_{2j}\, v_{2j+1}) = 6j-1 \quad 1 \le j \le n-1.$

$f_{lpl}^*(v_{2j-1}\, v_{2j+2}) = 6j, \quad 1 \le j \le n-1.$

$f_{lpl}^*(v_{2j}\, v_{2j+2}) = 6j+1 \quad 1 \le j \le n-1.$

Clearly f_{lpl}^* is an injection.

gcin of (v_{2j+1})

= gcd of $\{f_{lpl}^*(v_{2j-1}\, v_{2j+1}), f_{lpl}^*(v_{2j}\, v_{2j+1})\}$

= gcd of {6j-2, 6j-1}

= 1, $1 \le j \le n-1.$

gcin of (v_{2j+2})

= gcd of $\{f_{lpl}^*(v_{2j-1}\, v_{2j+2}), f_{lpl}^*(v_{2j}\, v_{2j+2})\}$

= gcd of {6j, 6j+1}

= 1, $1 \le j \le n-1.$

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $D_2(P_n)$ admits linear prime labeling.

Example 2.7 G = $D_2(P_4)$

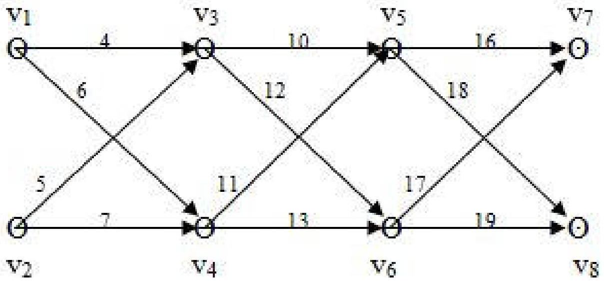

fig – 2.7

Theorem 2.8 Direct splitting graph of path P_n (n > 2) admits linear prime labeling.

Proof: Let G = $S'(P_n)$ and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,---,2n-1} by

$f(v_i) = i-1$, i = 1,2,---,2n

Clearly f is a bijection.

f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{2j}\, v_{2j+1}) = 6j-1, \quad 1 \le j \le n-1.$

$f_{lpl}^*(v_{2j-1}\, v_{2j+2}) = 6j, \quad 1 \le j \le n-1.$

$f_{lpl}^*(v_{2j}\, v_{2j+2}) = 6j+1, \quad 1 \le j \le n-1.$

Clearly f_{lpl}^* is an injection.

gcin of (v_{2j+2})

= gcd of $\{f_{lpl}^*(v_{2j-1}\, v_{2j+2}), f_{lpl}^*(v_{2j}\, v_{2j+2})\}$

= gcd of {6j, 6j+1}

= 1, $1 \le j \le n-1.$

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $S'(P_n)$ admits linear prime labeling.

Example 2.8 G = $S'(P_4)$

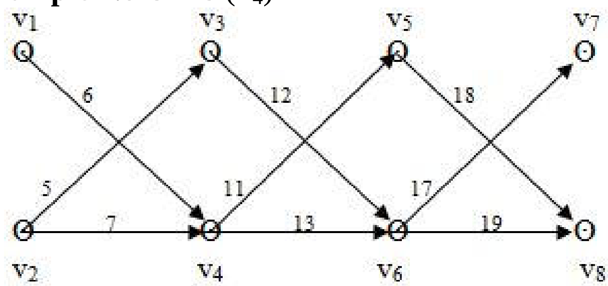

fig – 2.8

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